An Empirical Study on Fractal Features of China Stock Market

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***Abstract*—This paper utilizes R/S analysis method to investigate fractal features of Shanghai and Shenzhen Stock Exchange in China. The results prove that China stock market is a complex system with fractal features, and long-range dependence behaviour exists in its return sequence. Different from the traditional financial theory, the stock price has long-term memory behaviour, instead of random fluctuation.**

***Index Terms*—complex system; long-range dependence; stock market**

I. Introduction

The traditional financial theory always takes stock market research based on efficient market hypothesis (EMH) [1]. The theory proposes that the present price of stock has reflected all public information adequately, the future price is decided by new information, and the stock price can't be predicted according to given information. In today's capital markets, several important theoretical models, from modern portfolio theory of Markowitz, to capital asset pricing model of Sharpe, Lintner and Mossin, arbitrage pricing theory of Ross, and options pricing model of Black—Scholes, develop based on EMH or has close relationship with EMH. Generally, the above theoretical models are established based on three core assumptions, including rational investors, efficient market and random process, and the research is conducted under linear framework.

With the deep research of financial market, the phenomenon of size premium [2-3], value premium [4-5], momentum strategy profitability [6-7] and contrarian strategy profitability [8-9] has been found gradually. They are called Financial Anomalies deviating efficient market [10]. For the above abnormal phenomenon, the mainstream financial theories mostly stress the deviation of real financial market from balanced market, while the possibility that the financial market, as the complex system, performs the above characteristics inherently isn't considered.

The capital market is a system formed by listed company, exchange and investor. The input of system is information flow and capital flow, the output of system is mainly price and trading volume, and feedback effect exists between input and output. For transaction behaviour, listed company and exchange are relatively fixed and "dead" factors, while investor is flexible and "live" factor. Synthesize capital markets around the world, although the share price and volume sequence in different listed companies and trading mechanism are various, their macro statistical properties are similar. For example, the return sequence has characteristics of peak and fat-tailed distribution, return volatility cluster and long-term correction.

Therefore, the complexity of stock market is not only decided by listed companies and trading mechanism, but also involves various investors. Unlike the hypothesis of traditional financial theory, investors in actual market don't have complete information. Investors without complete information have different reaction modes and different prediction and decision-making mechanism. Every investor conducts the prediction and investment decision-making under already existed information, and then forming market price, the information included in market price influences the prediction and decision-making of investors conversely. Thus, the investor continually verifies and updates prediction rule according to market change and experience of winner to prompt the coevolution of prediction rule and market. In evolutionary process, the gather, differentiation and nonlinear responses to information of micro investors creates the complexity of entire capital market in macro level.

II. Experimental procedure

A. Fractal definition

The fractal is a powerful tool of depicting complex system. It is the emerging complexity science developed in the 1970s, which studies the self-similarity property of complex system, meaning the system with same or similar characteristics exists under different scale [11].

The fractal structure exists in financial market, and the price sequence at day, week and month has similar track. Therefore, the complexity of China stock market can be estimated by means of investigating fractal features, namely long-range dependence of China stock market.

The time sequence commonly used in economics and engineering is short-range correlation. For example, the self-correlation function of ARMA sequence shows exponential damping with respect to lag order. While the long-range dependence sequence is a kind of special time series, and its self-correlation decay speed is very slow, so that after long lag order, the remarkable correlation between two random variables before and after system still exists, and it will be tenable similarly in the certain time scale. It means the early history of system is important in deciding both current state and future state of system. Furthermore, the current state of system probably incudes the partial information of system evolution, and can influence the future [12].

Definition: if real number $\alpha \in (0,1)$ and constant $c_\rho > 0$ exist in steady sequence X_t, get

$$\lim_{k\to\infty} \rho(k)/[c_\rho k^{-\alpha}] = 1$$

Then the sequence X_t is called stationary process of long-term memory or long-range dependence, or stationary process of α in slow decay. The self-correlation of this process decays according to power law, significantly slower than common exponential decay time sequence.

For $\frac{1}{2} < H < 1$, $\rho(k)/[H(2H-1)k^{2H-2}] \to 1$, this process has long-range dependence. When H＝0.5, the correction being greater than first order will be zero, which is the characteristic of Brownian movement. The long-range dependence is a universal characteristic of self-similarity system, and self-similarity parameter is to depict long-range dependence [13].

B. R/S analysis method

In long-range dependence research, there are mainly 3 tools: R/S analysis, DFA method and spectral method. The paper will adopt R/S analysis method, which was firstly proposed empirically in the 1950s, and then Mandelbrot, one of founders of fractal, conducted the strict mathematical proof and development, thus making it to be a powerful tool for studying long-range dependence [14].

R/S analysis was put forward firstly by Hurst, who is a water scientist of England. His research was calculating the best water storage of the Nile reservoir based on observed time series of reservoir runoff. Hurst found that the random inflow sequence supposed usually was not random, on the contrary, the certain stable correlation behaviour exists in time scale of several years. He found that the inflow volume tended to “cluster”, meaning the inflow volume of successional several years was lower than average level, but inflow volume of the following several years was probably higher than average level. This clustering phenomena proved long-range dependence existed in the system. Hurst proposed a new statistical magnitude to identify the systemic non-random characteristic, namely Hurst exponent. Mandelbrot etc. proved this statistical magnitude was superior to traditional method of distinguishing correction, such as self-correlation function, variance ratio, etc. In financial market research, the particularly important point is that Hurst exponent has strong distinguished ability for random and non-random sequence when the real distribution is non-Gaussian distribution [15].

The specific steps of R/S analysis method are as follows:

(1) Define the return sequence of N in length $\{R_t, R_t = \frac{LnP_{t+1}}{LnP_t}\}$, and divide it to be continuous subintervals, in which the length is n. Remark each subinterval to $I_a, a = 1,\cdots,A$. And then each point of I_a can be expressed as $R_{k,a}, k = 1,\cdots,n; a = 1,\cdots,A$

(2) For each subinterval I_a in length of n, the mean value is calculated to $e_a = \frac{1}{n}\sum_{k=1}^{n} R_{k,a}$.

(3) For the single subinterval, calculate its cumulative mean deviation: $X_{k,a} = \sum_{i=1}^{k}(R_{i,a} - e_a)$, $k = 1,2,\cdots n$, and the sum of cumulative mean deviation sequence in the single subinterval $\{X_{1,a}, X_{2,a},\cdots, X_{n,a}\}$ is zero.

(4) Define the range of single subinterval to be $R_{I_a} = \underset{k}{Max}(X_{k,a}) - \underset{k}{Min}(X_{k,a})$, $k = 1,2,\cdots n$.

(5) Calculate the standard deviation of each subinterval R_{I_a}/S_{I_a}: $S_{I_a} = \sqrt{\frac{1}{n}\sum_{k=1}^{n}(E_{k,a} - e_a)^2}$.

(6) Calculate the average rescaling range of A subintervals for different partition length n:

$$(R/S)_n = \frac{1}{A}\sum_{a=1}^{A}\left(\frac{R_{I_a}}{S_{I_a}}\right)$$

For different partition length (namely different time scale) n, repeat the above calculation process, many average rescaling range values can be got. Mandelbrot proved the linear relation exits between log (R/S) and log (n).

$$Log(R/S)_n = a + H * Log(n)$$

Conduct the double logarithm regression to n and R/S, its slope is parameter of long-range dependence, namely Hurst exponent H.

H＝0.5 means the sequence is mutually independent in every scale, which is the characteristic of deterministic system; 0.5<H<1 means the sequence has relevant characteristic in every scale of self-similarity, namely the price change of day, week and month is related to future price change of day, week and month. It indicates if the price rises in prior period, the next period will continue to rise probably, and this correction (within self-similarity range) is irrelevant to time scale; if <H<0.5, the sequence will show inverse correlation in every scale.

In most real complex systems, the long-range dependence behaviour has time limitation since the self-similarity is bounded. When the influence of initial conditions in system disappears, the long-range dependence will disappear. In the time scale being more than long-range dependence duration, the system shows the irrelevant random behaviour. In the regression chart log(R/S)－log(n) of R/S analysis, it is easy to observe the sudden change of Hurst exponent at anywhere, and the corresponding N at sudden change is the time span of initial conditions information lost completely in system, namely the average orbital period of system.

III. Results and discussion

This paper takes day sequence of comprehensive index in Shanghai Stock Exchange from October 5, 1992 to September 30, 2015 as research sample, and calculates the returns ratio sequence of Shanghai composite index during the same term according to the formula $R_t = \ln(P_t - P_{t-1})$. The data source is Wind Database (Wind.com), and the data processing software is SPSS19.0.

A. Results of Shanghai composite index

Make the log(R/S)-log(n) line chart for Shanghai composite index (Figure 1), conduct regression, get the regression equation $\log(R/S)_n = -0.467 + 0.535\log(n)$, F statistical magnitude is 5611, T statistics of slope is 55.496, R^2=0.978, DW=1.95.

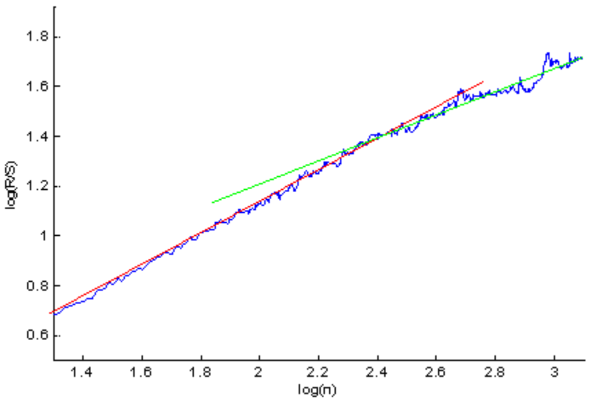

Figure 1. Regression line chart of Shanghai composite index

From the above computation, the estimated Hurst exponent of Shanghai composite index at all time scale is 0.535, which doesn't deviating the random range significantly. The further investigation is conducted to R/S line chart of Shanghai composite index, it can be found that the significant slope change exits in time scale of n=250. Therefore, the further segmented logarithm regression to data before and after n=250 is conducted, and the result is shown in Table 1.

TABLE 1
SEGMENTED R/S REGRESSION OF SHANGHAI COMPOSITE INDEX

Regression result	n<250	n>250
Slope	0.658	0.478
R^2	0.996	0.956
F statistics	6011.3	1252.6

The result of segmented regression indicates the correction behaviours of Shanghai composite index before and after n=250 are different significantly. Before n=250, H=0.6580, the significant long-range dependence exits in Shanghai composite index, while after n=250, H ＝0.4789, it extremely approaches to random behaviour. It indicates Shanghai composite index is a nonlinear system with persistent characteristic, and the average orbital period of system is 250 days. Afterwards, the long-range dependence disappears gradually, and the system tends to random fluctuation.

B. Result of Shenzhen component index

Make the log(R/S)-log(n) line chart for Shenzhen component index (Figure 2), conduct regression, get the regression equation $\log(R/S)_n = -0.481 + 0.608\log(n)$, F statistical magnitude is 9715, T statistics of slope is 38.254, R^2=0.993, DW=2.01.

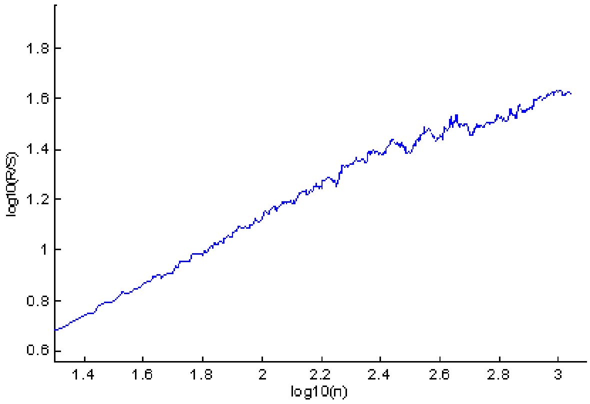

Figure. 2 Regression line chart of Shenzhen composite index

From the above computation, the estimated Hurst exponent of Shenzhen component index at all time scale is 0.608. The further investigation is conducted to R/S line chart of Shenzhen component index, it can be found that the inflection point appears in time scale of n=261. Therefore, the further segmented logarithm regression to data before and after n=261 is conducted, and the result is shown in Table 2.

TABLE 2
SEGMENTED R/S REGRESSION OF SHENZHEN COMPONENT INDEX

Regression result	n<261	n>261
Slope	0.686	0.504
R^2	0.989	0.963
F statistics	5270.9	756.2

The result of segmented regression indicates the correction behaviour of Shenzhen component index before and after n=261 are different significantly. Before n=261, H=0.686, the significant long-range dependence exits in Shenzhen component index, while after n=261, H ＝0.504, it extremely approaches to random behaviour. It indicates Shenzhen component index is a nonlinear system with persistent characteristic, and the average orbital period of system is 261 days. Afterwards, the long-range dependence disappears gradually, and the system tends to random fluctuation.

Ⅳ. CONCLUSION

The Hurst exponents of Shanghai Stock Exchange and Shenzhen Stock Exchange are higher than 0.5, indicating the long-range dependence behavior exists in the return sequence of China stock market, which means the stock yield moves to one direction continuously within a period of time, the revenue of current period rises, and then the revenue of next period will rise probably. Different from the opinion of traditional financial theory, the fluctuation of share price has the long-term memory behavior, unlike Brownian movement. Therefore, China stock market is the complex system with fractal structure.

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